

Essential Characteristics of Culturally Responsive and Sustaining Education in Mathematics Curriculum Materials: Sample Lessons

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Culturally responsive and sustaining education (CRSE) is an approach to education that seeks to advance educational equity by creating culturally affirming and inclusive learning environments, experiences, and outcomes for each student. CRSE recognizes students' cultural and other differences—including racial, ethnic, linguistic, gender, sexuality, and ability differences—as assets for teaching and learning. These differences can also serve as cultural capital, or knowledge, that can be leveraged in classrooms, schools, and districts to support students' development of positive identities and academic and social–emotional learning.

CRSE is informed by a constantly evolving cross-section of research and fields of study, including culturally relevant pedagogy, culturally responsive teaching, racial identity development, bilingualism, student agency, critical race theory, and social–emotional learning. At its heart, CRSE involves making substantive connections between students' cultural identities and lived experiences with the content and skills that they learn in school in ways that support cultural pluralism, not cultural assimilation.

Five Essential Characteristics

The reference guide *Essential Characteristics of Math Curriculum Materials That Advance Culturally Responsive and Sustaining Education* identifies the following five research- and evidence-based essential characteristics of math curriculum materials that advance CRSE:

1. Affirms racial and cultural identities
2. Cultivates agency and belonging
3. Integrates socially responsible, real-world contexts
4. Centers community knowledge
5. Promotes critical mathematical thinking and learning

This section articulates the core features of math curricula that embody these essential characteristics. Each characteristic has a link to a related sample lesson that provides greater detail about what it looks like to embed the respective characteristic and aligned features in the context of a mathematics lesson.

Teacher-Created Sample Lessons

The sample lessons were created in 2023 during a series of design labs with exemplary elementary, middle, and high school mathematics teachers from California, Florida, New York, and Texas. The labs were run by WestEd with the goal of creating sample lessons that demonstrate what culturally responsive and sustaining math education, aligned to the essential characteristics, could look like in supplemental math curriculum materials and in classroom teaching practice.

Using the five essential characteristics, the teachers researched and created sample lessons to illustrate selected features. Some of the samples were developed using open educational resources that teachers found online and adapted to align to the characteristics; others were wholly created by teachers. Participants were asked to choose at least one characteristic to align to their sample lesson. They then engaged in peer evaluation of the created lessons using criteria aligned to the definition of CRSE. Feedback was also provided to participants by content experts in the field. The lessons are aligned to both mathematics content standards (e.g., Common Core State Standards) and the Standards of Mathematical Practice.

Audience for These Lessons

While the main audience for these sample lessons are K–12 math teachers, the lessons are also intended to support teacher professional learning and may be valuable to those charged with training math teachers. Additionally, these sample lessons can be used by educators and education leaders who are responsible for creating and purchasing curriculum materials to inform their work to advance culturally responsive and sustaining math education.



Essential Characteristic 1: Affirms Racial and Cultural Identities

Sample Middle School Lesson—Characteristic 1

Math curricula affirm the cultures, languages, and identities of all students and community members.

- Curriculum materials value student, family, and community cultural and linguistic practices and are used as vehicles for acquiring and demonstrating knowledge and skills.
- Students' and families' racial and other identities are reflected and validated in all materials.
- Curriculum materials consist of accurate portrayals of all cultures and identities and are free of all forms of bias and stereotypes.



Essential Characteristic 2: Cultivates Agency and Belonging

Sample High School Lesson—Characteristic 2

Math curricula support the cultivation of a community of learners and build student agency.

- Curriculum materials consist of open-ended, problem-posing structures to normalize centering students' individual and collective expertise in an authentic and meaningful way.
- Curriculum materials cultivate a community of learners in a way that builds students' ability to connect across cultures and identities.
- Curriculum materials include approaches and strategies that leverage and grow students' social–emotional competencies.



Essential Characteristic 3: Integrates Socially Responsible, Real-World Contexts

Sample High School Lesson 1—Characteristic 3

Sample High School Lesson 2—Characteristic 3

Math curricula build student capacity to understand and address historical and contemporary injustice.

- Curriculum materials provide opportunities to acquire and apply mathematical skills and concepts to further students' understanding of the relationships between race, culture, identity, power, and privilege and the historical and current systems that create and sustain inequality.
- Students learn mathematical concepts and apply critical thinking practices to connect them to real-world situations at the local, national, and global levels.
- Curriculum materials prompt students to take collective action to address local, national, or global issues of injustice.



Essential Characteristic 4: Centers Community Knowledge

Sample Elementary School Lesson—Characteristic 4

Math curricula challenge dominant narratives about math and school, elevating historically marginalized voices.

- Students learn about the histories, lived experiences, and world views of people who have been historically excluded from dominant narratives about mathematics and schooling.
- Curriculum materials challenge traditional ways of knowing or doing math (e.g., right/wrong binary, base-10, emphasis on individual achievement) by providing students opportunities for open-ended problem-solving, social connection, and connecting math to their context.
- Curriculum materials provide prompts, supports, and resources for teachers to validate student and community mathematical assets.



Essential Characteristic 5: Promotes Critical Mathematical Thinking and Learning

Sample High School Lesson—Characteristic 5

Math curricula hold high expectations for student learning, supporting the development of positive math identities.

- Curriculum materials are aligned with relevant state and/or national standards and ensure that students will acquire the knowledge and skills needed for academic success.
- Students have multiple, diverse opportunities to display and communicate their thinking.
- Students have the opportunity to reason, problem-solve, and make real-world connections.
- Students are encouraged to talk about their learning and provided with effective scaffolds.
- Curriculum materials provide opportunities to validate students' effort, not just ability.
- Curriculum materials encourage student questioning and ownership of their learning.

Appendix A.
Sample Mathematics Lesson for Essential Characteristic 1: Affirms Racial and Cultural Identities

This lesson was created by a teacher during a design lab focused on culturally responsive and sustaining education (CRSE). It focuses on a core feature of Essential Characteristic 1, described in table A1.

Table A1. Overview of Essential Characteristic 1

	Details
Essential characteristic	Characteristic 1: Affirms racial and cultural identities
Description of essential characteristic	Math curricula affirm the cultures, languages, and identities of all students and community members.
Core feature that this lesson relates to	Students' and families' racial and other identities are reflected and validated in all curriculum materials.

Table A2 lists the mathematical standards to which this lesson aligns.

Table A2. Mathematical Standards Related to This Lesson

Component	Details
Source	Common Core State Standards
Domain	Ratios
Mathematical standard	6.RP.A.1: Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. 6.RP.A.3: Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

Lesson Title: Income Inequalities for College Athletes

Lesson Goals

In this lesson, students will examine inequalities in sports through the use of ratios.

Goals of this lesson:

- Promote student engagement by giving students time to be curious and to explore and by making the math lesson fun, active, and meaningful.
- Make the content relevant to students by connecting problems, especially word problems, to students' lived experiences and communities and by including opportunities for projects that involve family and community engagement.
- Elevate all voices and promotes diverse perspectives.
- Prompt students and teachers to self-reflect.
- Center higher order and critical-thinking skills.
- Provide opportunities for student agency.

Lesson Setup

Table A3 provides some key information to help teachers set up and carry out the lesson.

Table A3. Lesson Setup and Logistics

Aspect of Lesson Setup	Details
Lesson resources	• Slides for the lesson
Key vocabulary	• Income inequalities • Average
Prior knowledge	• Rational number operations • Reading a table • Reading a line graph • The concept of monthly average rent

Aspect of Lesson Setup	Details
Strategies for forming groups	• Consider using random grouping strategy to create pairs of students. • **Note:** The Orange Student scenario can be used for students who need a challenge.
Potential setup	Different stations can be used for this lesson: • Station 1: Complete the table • Station 2: Create an equation • Station 3: Create a graph

Lesson Procedures: Day 1

Task

Warm-Up (10 minutes total)

Students will launch into a warm-up discussion using the following "Would you rather …?" scenario:

1. Select an option and explain the pros/cons:
   a. Option 1: Go to a college/university near your family's home?
   b. Option 2: Go to a college/university out of state?
2. Students are given 2 minutes of independent think time. (2 minutes)
3. Class does a whole-group share-out. (5 minutes)
4. Teacher talks about the pros/cons for each option to give students a better understanding. (3 minutes)

Task

Explore (15 minutes total)

Students will launch into looking at the following four different scenarios about college students who are athletes and need to work part-time to support living expenses:

- **Red Student (1 adult):** You are a Filipino American male who just graduated from high school and needs to move out on your own. You found a job making minimum wage for a nontipped employee in San Diego, $15.50 per hour as a cook at a nearby restaurant. You are also attending Palomar College while playing baseball. You work 30 hours a week.
- **Green Student (1 adult):** You are a young, single, White female. You work part-time at Starbucks and earn a little over the minimum wage of $17.50. Your parents have offered to pay 100% of your off-campus rent. You are attending University of California, San Diego while playing volleyball. You work 20 hours a week.
- **Purple Student (1 adult):** You are a Hispanic male who just got accepted to San Diego State University on a soccer scholarship that will pay for 50% of your school tuition ($34,000), not including housing. You will be working part-time with your uncle on the weekends, being paid $20.50 for 16 hours. You also support your parents.
- **Orange Student (2 adults):** You are an African American couple that is completing their second year at California State University, San Marcos. You both are college athletes: tennis and football. You both work 14 hours and get paid $16.50 at the library. Financial aid covers both tuitions.

1. Assign students to pairs.
2. Pairs read through each scenario and discuss what they notice and wonder about each student in each scenario.
3. Pairs discuss which student they would rather be if they were a college athlete who needs to work to pay living expenses. Once they have selected a student, they list out two reasons why they picked that option.
4. After partner conversation, ask each pair to select a reporter by identifying the person whose birthday is closest to that day.

Task

Whole Class Debrief: Four Corners (15 minutes total)

Students will try to convince peers that their student athlete has the most advantageous situation.

1. Assign a scenario to each corner of the room: one for red, green, purple, and orange.
2. Ask the random reporter for each pair to move to the corner of the room that represents the student athlete they would choose to be. Once there, they have 3 minutes to discuss with each other why they picked that student athlete.
3. After 3 minutes, ask each corner to identify whose birthday is closest to that day—that person will serve as the random reporter for their corner and student athlete. Tell these four reporters that their job will be to convince their classmates to choose their student athlete and scenario, even if it's not the one they chose initially.
4. Each corner takes turn reporting out and trying to convince their classmates to choose their student athlete. Reporters may "phone a friend" in the audience to add other persuasive details if needed.

5. Some anticipated arguments students may offer include the following:
   a. "We notice that this student only works this many hours, which leaves more time for studying and sports."
   b. "We notice the Green Student is supported by her parents, which is an advantage."
   c. "We notice the Green Student makes the most out of all the students."
   d. "We notice that the Purple Student scenario has two adults and they both have to work."
6. After all corners have presented, ask all students to move to the corner of the student athlete they would ultimately choose to be based on the arguments they have heard.

Task

Closing (5 minutes total)

On their exit cards, each student records which student athlete they selected to be, with two reasons to justify their selection

Lesson Procedures: Day 2

Task

Warm-Up (5 minutes total)

1. Students make individual predictions about what the numbers 2,610 and 2,875 represent. (1 minute)
2. They share their predictions with an elbow partner. (1 minute)
3. The teacher uses random calling to select three students to share their predictions with the whole class. (2 minutes)
4. The teacher reveals what the numbers mean: the difference between rent and mortgage costs in San Diego. (1 minute)

Task

Explore (15 minutes total)

1. Students return to the pairs they worked in during the previous class.
2. Direct students to review the scenario for their assigned student athlete from the previous lesson.

3. After review, tell them that they will be determining whether or not their student athlete can afford to live in San Diego by calculating whether or not they can afford rent or a mortgage in addition to other living expenses.
4. First, they will calculate each student athlete's weekly income using the information included in the scenario. They will fill in a table with how much their student athlete would earn weekly based on a certain number of hours worked.
   a. How many hours would they have to work a week in order to be able to afford housing monthly?
   b. What factors play into them being able to afford housing?
   c. What other expenses would they need to consider?

Task

Whole Class Debrief (20 minutes total)

Use the following questions as prompts:

» What did you notice about your student athlete's income?

» Can each student afford to live in San Diego?

» What factors play into your student being able to afford housing?

» What other expenses would your student need to consider? Project this Cost of Living in San Diego webpage to identify average costs for nonhousing monthly expenses in San Diego.

» What did you notice about each student athlete's race/ethnicity and family situation and how that was (or was not) connected to their being able to afford living expenses?

Task

Closing (5 minutes)

On their exit cards, have students respond to the following prompt: "The student athletes we studied faced inequities in terms of their race/ethnicity, financial support, and sport played." Do you agree or disagree with this statement? Explain your reasoning.

Elevating Culturally Responsive and Sustaining Education in This Lesson

» By examining difference in race/ethnicity, family background/financial support from parents, and sport played, students can conclude that all contribute to the amount and type of inequities that student athletes face.

» Cultural elements that can be elevated into the whole group debriefing both days include whether students are expected to

- live at home or independently while attending college and
- receive financial support from their parents while attending college or whether they need to provide financial support to their families.

» Try not to shy away from discussing these cultural differences and how they align with the background and experiences of the students in your classroom (e.g., Do they plan to attend college? Would they expect to do so while living at home or independently? Do they anticipate having to work while attending school?)

» It is important for the teacher to consider how their own experiences attending college and working and living independently while doing so may align or diverge with students' expectations for their own futures. Teachers cannot convey any judgment for cultural differences in expectations for students' college ambitions when expectations do not align to their own.

Appendix B.
Sample Mathematics Lesson for Essential Characteristic 2: Cultivates Agency and Belonging

This lesson was created by a teacher during a design lab focused on culturally responsive and sustaining education (CRSE). The lesson aligns to Essential Characteristic 2, which is described in Table B1.

Table B1. Overview of Essential Characteristic 2

	Details
Essential characteristic	Characteristic 2: Cultivates agency and belonging
Description of essential characteristic	Curriculum materials consist of open-ended, problem-posing structures to normalize centering students' individual and collective expertise in an authentic and meaningful way.
Core feature that this lesson relates to	Math curricula incorporate open-ended questions within the math curricula that give students the opportunity to analyze proofs and theories.

Table B2 lists the mathematical standards to which this lesson aligns.

Table B2. Mathematical Standards Related to This Lesson

Component	Details
Source	Florida B.E.S.T. Standards
Domain	Geometry (Population Density using area)
Mathematical standard	MA.912.GR.4.4 Solve mathematical and real-world problems involving the area of two-dimensional figures. Instruction includes concepts of population density based on area.

Lesson Title: Population Density Using Area

Lesson Goals

This lesson examines the intersection between population density and overpopulation using the mathematical concept of area.

Mathematical goals of the lesson:

» Solve real-world problems that involve population density.

» Make the connections between population vs. mass and area vs. volume.

Social justice goals of the lesson:

» Understand what overpopulation looks like and how it affects resources.

» Learn about population density and, in turn, understand how population affects an environment and the resources available.

Lesson Setup

Table B3 provides some key information to help teachers set up and carry out the lesson.

Table B3. Lesson Setup and Logistics

Aspect of Lesson Setup	Details
Lesson resources	• Materials: Poster board; art supplies (pencils, markers, colored pencils); access to print and online resources; one laptop per small group
Lesson duration	• No longer than two 50-minute class periods
Key vocabulary	• Population density • Area • Density
Prior knowledge	• Conversions in U.S. and metric systems • Calculating area • How to display data in tables and scatterplots

Lesson Procedures

Warm-Up (5 minutes)

1. Students individually brainstorm answers to the following questions and then discuss them with an elbow partner:
   a. What is density?
   b. What do you know about calculating density?
2. Discuss answers as a whole class:
   a. What is density?
   b. What do you know about calculating density? (It is the amount per unit size; a number you have probably calculated in science class.)

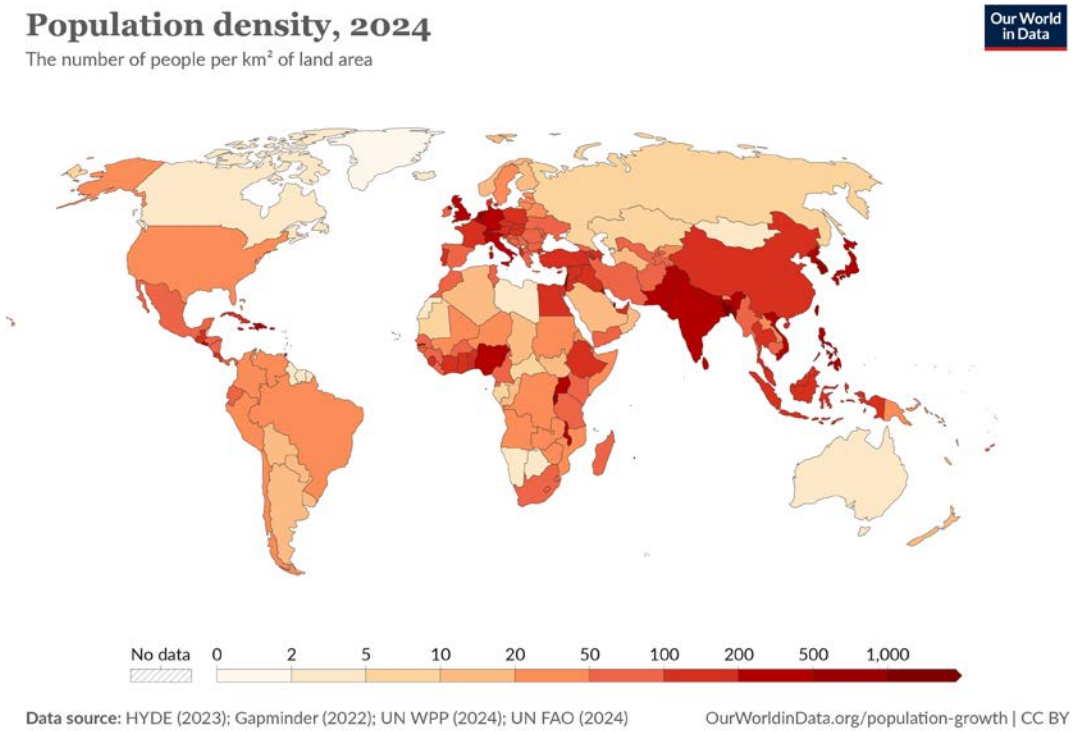
Explore (20 minutes)

1. Ask students: "Have you ever been to a crowded store during the holidays? It can feel like the whole population of your town or city is crammed into a tiny area. For a lot of people in the world, this is what they experience every day. Depending on their location, people experience a variety of different population densities. Today, we are going to learn about the concept of population density, so that we can understand how population affects an environment and the resources available."
2. Share definition of population density and formula for calculating it:
   a. Population density is a measurement of population per unit land area (square mile [sq mi] or kilometer [km]).
   b. Population density is calculated by dividing the population (number of people in an area) by the sq mi or km in that area, which yields the number of people per sq mi (one unit).
3. Project the map below and ask students what they notice and wonder about the population density in different parts of the world.
4. Share: "The world's population is around 8,000,000,000, and the Earth's land area is 58,000,000 sq mi. So, the population density of Earth is 140 people per sq mi. If we exclude Antarctica, the population density rises to 150 people per sq mi. This might not seem that high, but 95 percent of the world's population is concentrated on just 10 percent of the world's land."
5. Share: "Let's calculate the population density of some places we are interested in. We'll start with our county and state, and then let's make a list of other places to explore, like the states or countries our families are from or places we have visited in the past." [Brainstorm a list on screen with students to ensure they research areas that are relevant to them.]
6. Assign students to small groups of three to four and provide each with a laptop, chart paper, and markers. Their task will be to calculate the population density of the brainstormed areas using the following resources:
   a. For U.S. locations, use this QuickFacts web page from the U.S. Census Bureau.

i. Enter the state, county, city, and town name and then select "Population estimates, 2020" from the drop-down menu. Record that number, then select "Land area in 2020" and record that number. Use these numbers to calculate the population density of the selected location. **Students should also explore the "Income and Poverty" drop-down menu to examine connections between population density and income.**

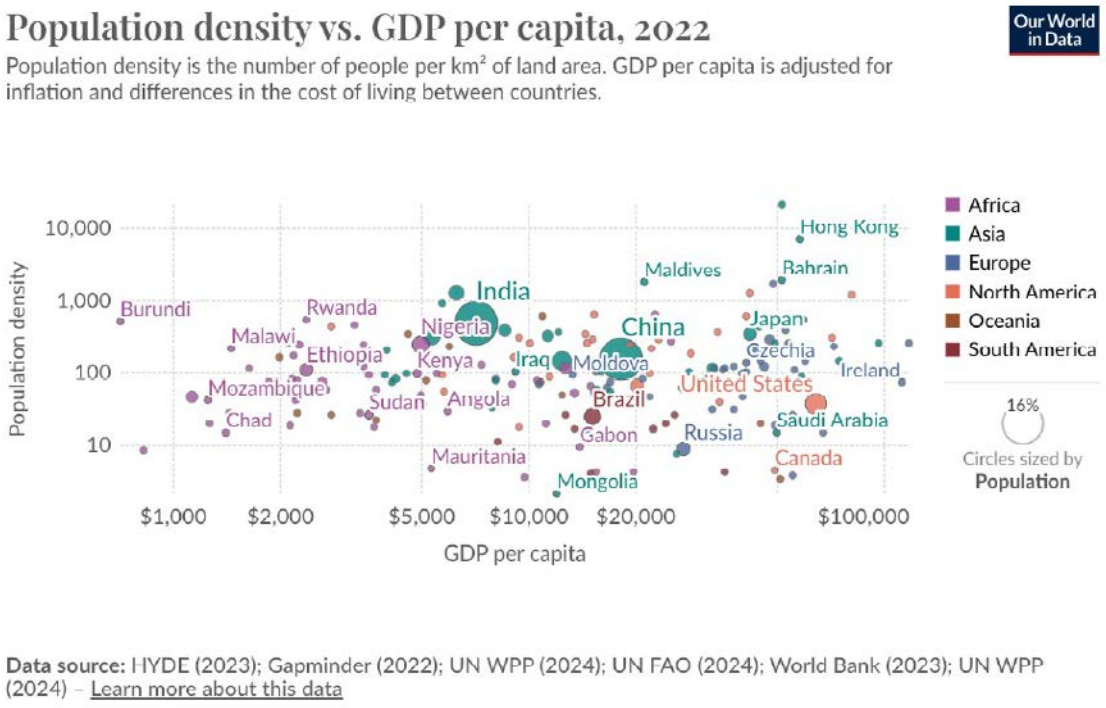
b. For international locations, students can use the "data-for-countries-etc-by-year" tab of this Population Dataset spreadsheet to identify the population in this calendar year and then search online for the area in square miles to calculate the population density.

7. Have students work together to create a poster (or slides) with a table displaying the population densities calculated for the brainstormed locations in order from most to least dense as well as a scatter plot that displays population on one axis and area on the other.



Whole Class Debrief (15 minutes)

1. What does the population density of a country or state tell you?
2. How were the population densities of our town, state, and the foreign country similar and different?
3. How do you think the population density of our town or state affects our daily lives?
4. In the U.S., you got to explore the connection between population density and income/poverty. What did you notice?
5. Let's look at a scatter plot that shows the population density of countries compared to the gross domestic product (GDP), a measure of relative wealth or poverty. What do you notice? Wonder?
6. What can we conclude about the connection between population density and income?
7. Do you think within the next 10 years we could possibly run out of area to house the growing population comfortably?
8. Do you think that other countries should adopt China's idea of curbing the population through restricting the birth rate?



Closing (5 minutes)

Exit card: Some areas of low population density, such as Alaska, offer financial incentives for people to move there. Would you accept money to move to a less populated area? If so, how much? Explain your rationale using what you learned about population density today.

Elevating CRSE in This Lesson

- Having students examine the population density of their own community and other places they have selected makes the lesson more meaningful and relevant for students.
- Elevate other related issues into the whole group discussion (e.g., solving the problem of homelessness; addressing the lack of affordable housing, which drives up the cost of rent; determining who is most impacted by rising prices) in order to prompt further critical thinking around real-world issues related to population density.

Appendix C.
Sample Mathematics Lesson for Essential Characteristic 3: Integrates Socially Responsible, Real-World Contexts

This lesson was created by a teacher during a design lab focused on culturally responsive and sustaining education (CRSE). The lesson aligns to Essential Characteristic 3, which is described in Table C1.

Table C1. Overview of Essential Characteristic 3

	Details
Essential characteristic	Characteristic 3: Integrates socially responsible, real-world contexts
Description of essential characteristic	Math curricula build student capacity to understand and address historical and contemporary injustice.
Core feature that this lesson relates to	Curriculum materials provide opportunities to acquire and apply mathematical skills and concepts to further students' understanding of the relationships between race, culture, identity, power, and privilege and the historical and current systems that create and sustain inequality.

Table C2 lists the mathematical standards to which this lesson aligns.

Table C2. Mathematical Standards Related to This Lesson

Component	Details
Source	Texas Essential Knowledge and Skills—High School (Advanced Placement Statistics)
Domain	Advanced Quantitative Reasoning AQR(4)(A)
Mathematical standard	Use a two-way frequency table as a sample space to identify whether two events are independent and to interpret the results

Lesson Title: Describing Categorical Data—Race and Policing

Source attribution: This lesson is published by Skew the Script and is made available through this Creative Commons license.

Lesson Goals

This lesson uses advanced quantitative reasoning to examine racial disparities in policing.

Lesson Setup

Table C3 provides some key information to help teachers set up and carry out the lesson.

Table C3. Lesson Setup and Logistics

Aspect of Lesson Setup	Details
Lesson source	Skew the Script (Lesson 1.2 Describing Categorical Data)
Lesson resources	All materials are provided in this appendix. This lesson can also be accessed by students in class or at home via Desmos if this modality is preferred.
Lesson duration	• Two 50-minute class periods
Key vocabulary	• Marginal distributions • Side-by-side bar graphs • Conditional distributions • Segmented bar graphs • Describing associations
Prior knowledge	• Calculating percentages • Reading graphs/charts • Dependent and independent variables

Lesson Procedures

Day 1: Investigate—"What Can Data Show Us About Policing and Race?"

Important note: Assess students' comfort level with discussing policing and race in a confidential way before engaging with this lesson. If students have had traumatic experiences with law enforcement officers, this lesson could likely retrigger them. The risk of activating trauma outweighs the effort of coming up with a new lesson on the same learning objective. There are other options that teach the same mathematical concepts with other content (Version B: Admissions and Version C: Basketball).

Warm-Up (5 minutes)

1. Ask a student volunteer to explain the concept of "stop and frisk" before projecting the definition.
2. Project the data on 2020 police stops in New York City (NYC). Direct students to discuss what they notice and wonder about the data with an elbow partner.
3. Use random calling to solicit responses from students. As needed, discuss or note the following:
   a. That "Yes" means that the person stopped was charged with a criminal offense and "No" means they were not
   b. Why certain groups (e.g., Middle Eastern, Native American) were excluded from the data due to the size of their sample

Explore (35 minutes)

Overview

1. Tell students that over the next three lessons they will explore the connections between policing and race in order to determine if NYC's "Stop and Frisk" policy unfairly target Black and Brown people.
2. Share Mayor Bloomberg's quote. Then tell students that they will create and analyze marginal distributions, side-by-side bar graphs, conditional distributions, and segmented bar graphs over the next two lessons in order to decide if Mayor Bloomberg's claim is supported by the data.

Marginal Distributions

3. The first step in determining the racial distribution of civilians stopped by the police is to create a two-way table that displays two categorical variables.
4. Have students discuss the following with a partner and then record on their partner worksheet (each pair only gets one sheet and must take turns recording answered to ensure they work interdependently): "What are the two variables described in the table above?" (See answers in blue on teacher key.)

5. Say: "Now that we know the variables, we need to calculate
   - the row totals for Yes or No reported offenses,
   - the column totals for each racial group, and
   - the grand total.

   Work with your partner to do so."
6. After providing time for pairs to calculate totals and record them on their shared worksheets, have students check their answers with those displayed on screen. Note that the row and column totals should both match if their calculations are correct.
7. Direct students to discuss with their partners: "Who do police tend to stop?"
8. Explain that in order to answer that question, we need to calculate the marginal distribution or break down the data for one variable—in this case, by race of civilian. So, for this, we will focus only on the columns and the grand total, not the rows.
9. Model how to calculate the distribution for Asian civilians. Then direct students to work with their partners, alternating who does the calculations and who records them on their worksheet, for the three remaining racial groups.
10. After providing time for pairs to calculate totals and to record them on their shared worksheet, have students check their answers with those displayed on screen. Then ask them to discuss the following with their partner: "Which racial groups get stopped by police most frequently?"

Side-by-Side Bar Graphs

11. Project the percentages of the NYC population vs. police stops. Ask students to discuss the following with their partner: "How does this distribution compare to the overall population of that racial group in NYC? What do you notice about the percentages?"
12. Direct students to look at the side-by-side bar graph on their shared worksheet. Say, "We know that bar graphs must include a title, tick marks on the y-axis, and labels for both axes. What does this graph also include that is not required but is helpful?" (Answer: "percentages for each bar.")

    "Now look at the bar graph on your worksheet. Compare the racial distribution of stops to the racial distribution of New York City. What do you notice? What do you wonder?" (Answer: "Police tend to stop Black civilians at a disproportionately high rate. They are the only group who are stopped at a higher rate than the rate we'd expect if randomly sampling among the population.")

Close (10 minutes)

1. Say: "So, after today we know that police stop Black civilians more than any other racial group, but we don't yet know if it's because they commit more crimes than other groups. We'll figure that out tomorrow."

2. Individual exit card: "Before we leave, let's see if you can interpret another marginal distribution table on your own this time. This one displays two variables:
   a. who gets called back for a second job interview, and
   b. whether their name is commonly considered a Black name or White name.

   Use the data to explain the connection between these two variables. Be sure to focus on the data presented in the rows for now, not the columns."

Day 2: Complete Investigation and Use Data to Make an Argument

Explore (35 minutes)

Overview

1. Say: "Yesterday, we calculated marginal distributions of stops in NYC by racial group and examined side-by-side bar graphs of these rates. Today, we will continue exploring the connection between race and policing using conditional distributions and segmented bar graphs. We'll then use both to describe associations between the two variables."
2. Say: "You'll be working with your same partner from yesterday during your shared worksheet. You will again take turns making calculations, creating data displays, and answering questions on your shared sheet."

Conditional Distributions

3. Say: "We will use the same table that we started with yesterday, but this time, we will be focusing on columns instead of rows."
4. Model how to calculate the conditional distribution for Asians—the percentage for the "Yes," or charged with a crime category, and the "No," not charged category.
5. Say: "Taking turns with your partner, calculate the 'Yes' and 'No' conditions for the other racial groups using the table columns."
6. After providing time for pairs to calculate totals and record them on their shared worksheet, have students check their answers with those displayed on screen. Then ask them to discuss the following with their partner: "Which racial groups get charged with a crime most frequently?"

Segmented Bar Graphs

7. Ask students, "How can we visualize this data?"
8. Model how to create a segmented bar graph for the Asian civilian stops data.
9. Say: "Taking turns with your partner, create a graph with segmented bars to represent each racial group on your shared worksheet, Make sure to include title, tick marks, and labels."

10. After providing time for pairs to calculate totals and record them on their shared worksheet, have students check their answers with those displayed on screen. Then ask students to discuss the following with their partner: "How would you describe the crime rate among racial groups? Do you think there is evidence of an association between crime rates and race?"

Describing Associations

11. Say: "Now that we have created our graph, we will use it to determine if there is an association between race and crime rates. To answer that question, we'll need to
    - make a claim,
    - support the claim with data, and
    - include context using words like 'higher' or 'lower' when comparing variables."
12. Say: "Let's try one together: Overall, the crime rates were fairly similar among each racial group, yet there were some noticeable differences especially between Asian and White civilians."
13. Have students read the association and note its parts—the claim, the context, and the comparison between variables—in this case, racial groups.
14. Say: "Now work with your partner to answer the question on your shared worksheet: 'Do the data suggest an association between race and crime rates? Explain.' Create an association to answer this question that has all three parts."
15. Provide time for each pair to get up and share their associations with other pairs. Then debrief as a whole class about which associations students found most compelling and why.

Debrief

16. Say: "Let's return to Mayor Bloomberg's argument that more crime happens in neighborhoods with more minoritized racial populations. Based on the data we looked at today, do you agree with his argument. Why or why not? Discuss with your elbow partners."
17. Poll the students about their answers to the question in step 16—have students raise their hands if they agree and then raise their hands if they disagree. Ask each side to provide the data that supports their argument.
18. Share update: "Based on the data, Mayor Bloomberg retracted his statements and no longer supports Stop and Frisk."
19. Time permitting, engage in a whole class debrief about the following:
    - How can politicians use data to sway the public opinion?
    - Is all data representative of the truth?
    - What different types of data should citizens look for to make informed decisions?

Close (10 minutes)

1. Individual exit card: The AP Statistics exam will test your ability to make associations using marginalized and conditional distributions. Complete this table and then use the data to answer the questions about it:

Two hundred students were asked about their favorite sport. Of the 200 students surveyed, 98 were female. Some of the results are shown in the following two-way frequency table.

	Favorite Sport				
Gender	**Football**	**Baseball**	**Basketball**	**Soccer**	**Total**
Female			36	12	
Male	38	19			
Total	64			36	

2. Complete the table.
3. Which sport is the most popular amongst students? Which is the least popular? How do you know? Create one type of graph to represent your response.
4. Which sport is popular among females? Which sport is popular among males? Create one type of graph to represent your response.

Elevating CRSE in This Lesson

The following text is reproduced from the web page Teaching Math on Civic Topics: A Productive and Nonpartisan Approach, authored by China Cardriche and Dashiell (Dash) Young-Saver. This web page's content is published by Skew the Script and made available through this Creative Commons license.

Whether we like it or not, talking about important issues often involves talking about math. During the COVID pandemic, we saw doctors use statistics to advocate for national mask mandates, while governors used the same data to argue that individual liberty outweighed the risk. As protests occurred across the country, we heard colleagues use data as evidence to support Black Lives Matter or as a way to shift the focus to black-on-black crime. At home, we've heard argument after argument end with the question: "Well, where's the data on that?"

Our world is flooded with data, statistics, and numbers. We have more information at our disposal than ever before. Yet, our discourse often devolves due to bad mathematical reasoning: biased samples, correlation/causation fallacies, misleading denominators, and flimsy quantitative arguments appear almost everywhere we turn. To heighten public discourse, we need to prepare our future citizens to use rigorous quantitative thinking—to discern the well-evidenced claims from a sea of misguided conclusions.

When students bring real-world issues into their math class, we often tell them to leave it at the door. Socially relevant math takes the opposite approach. Instead of keeping real-world conversations out, we invite them in. Then, we ground the discussion in the scrupulous and common language of math. This orientation towards math instruction doesn't mean teaching with a political bias. In fact, it's the opposite: It prepares students to think critically, evaluate contrasting claims, and identify bias through mathematical reasoning. The only "political agenda" is to heighten political discourse. Importantly, we've found that this way of teaching authentically engages students, which also helps them understand the math itself at a deeper level.

We acknowledge that teaching socially relevant math can feel daunting. To tackle these challenges effectively and responsibly, we recommend the following practices.

Set and Maintain Strong Discussion Norms

1. Strong discussion norms are critical for creating a classroom environment in which students trust one another to agree, respectfully disagree, and move conversations/learning forward.
2. View our recommended discussion norms. We also recommend having students contribute further discussion norms to this list to build investment and to make your class more student-centered.
3. Maintaining discussion norms is as important as setting discussion norms. Appoint student leaders to hold their peers accountable to these norms.

Share Your Nonpartisan Intentions

» During curriculum planning meetings, in your course syllabi, and on parent–teacher night, make your intentions clear: Although we may analyze data on sociopolitical topics, our goal is to have fair, balanced, and data-informed discussions. There is no political agenda, other than heightening students' political discourse through data analysis and critical thinking.

» Make a copy of our letter to parents, which describes Skew The Script's lessons and nonpartisan curriculum approach. We recommend editing the letter to make it your own, then sending it out to parents at the beginning of the school year.

Come to Conversations Prepared

» Because you will discuss topics that are passionate for students, they will have many questions about the context of the data, the math involved, and the types of conclusions that can be drawn. Come prepared to answer these questions or to position students to answer these questions themselves.

Position Student Voice at the Center of Your Classroom

» Classrooms are about student learning and student voice; yet, too often, teachers make room for only their own voice to be heard. Allowing student voices to lead the classroom creates a more trusting, open, and impactful learning environment.

» There are many structures teachers can use to leverage student voice. At the outset of the year, have students generate discussion norms. During the year, ask students for topics of discussion, around which you can plan lessons. By the end of the year (oftentimes after standardized testing), have students collaborate to lead lessons on topics of their choosing.

Value Personal Experience Alongside Data

» Data often provides good evidence for making inferences about social issues. We also know that data can be used to mislead and, if not used cautiously, can lead us to incorrect conclusions. Make sure you know—and that students know—these simultaneous truths.

» Data can never invalidate personal experiences. All trends have variability and, in most cases, data has nothing to say about the truth of an individual experience.

Name: _____

AP Statistics Handout: Lesson 1.2

Topics: marginal & conditional distributions, bar graphs, association

New York City's **Stop and Frisk** program allows police to stop people on the street and search them for weapons or contraband. During a stop, if police find evidence of criminal items or activity, they report a criminal offense. Then, they can make an arrest or issue a summons. NYPD officers record detailed data[1] on every stop. Below are data from all 2020 stops among the most common racial groups.[2]

Police found & reported criminal offense?

Race of Stopped Civilian

	Asian	Black	Hispanic	White
Yes	77	2431	824	352
No	142	3765	1226	504

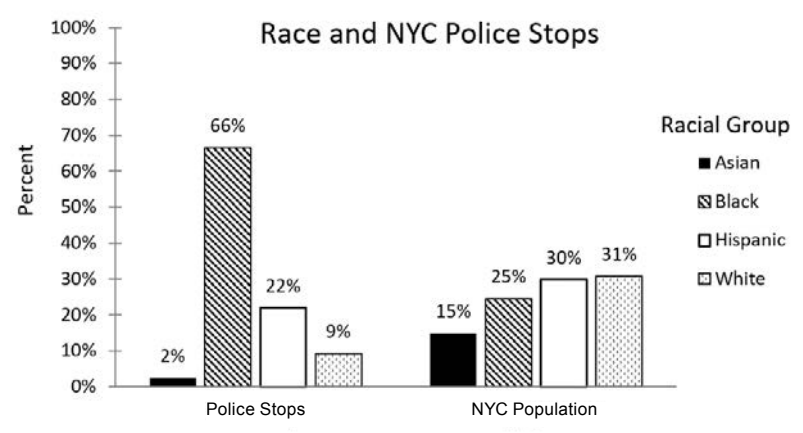
Two-Way Table: A table of counts describing two _____.

1. What are the two variables described in this table?

1 This NYPD data is publicly available on the NYC.GOV website: https://www1.nyc.gov/site/nypd/stats/reports-analysis/stopfrisk.page
2 Note: Groups not included (Middle Eastern, Native American, Not Reported) did not have a high enough sample size for this analysis. So, all percentages in this lesson are calculated only among the groups listed in the table. Data for other groups can be found in the NYC.GOV data file.

Marginal Distributions

2. Find the racial distribution of civilians stopped by police.



3. What feature of this **side-by-side bar graph** is <u>not</u> needed to satisfy "title, tick, tick, label, label, label"?

4. Compare the racial distribution of stops to the racial distribution of New York City.[3] What do you notice? What do you wonder?

3 NYC population data from 2020 American Community Survey (U.S. Census Bureau)

Conditional Distributions and Associations

Former NYC Mayor Michael Bloomberg, who expanded Stop and Frisk, speaking about the program (Aspen Institute, 2015):

> *"People say, 'Oh my God, you are arresting kids for marijuana who are all minorities.' Yes, that's true. Why? Because we put all the cops in the minority neighborhoods. Yes, that's true. Why'd we do it? Because that's where all the crime is."*

Today, advocates for Stop and Frisk make similar claims. Are these claims supported by current data? Let's investigate.

5. Calculate the crime rates found among each racial group.
   Display this information using a **segmented bar graph**.

6. Do the data suggest an association between race and crime rates? Explain.

   Note: What are the three key components you should mention when describing an association?

7. Are Mayor Bloomberg's comments supported by this data? Explain.

AP Statistics Handout Key: Lesson 1.2

Topics: marginal & conditional distributions, bar graphs, association

***All solutions and teacher notes in blue and preceded by an asterisk**

New York City's **Stop and Frisk** program allows police to stop people on the street and search them for weapons or contraband. During a stop, if police find evidence of criminal items or activity, they report a criminal offense. Then, they can make an arrest or issue a summons. NYPD officers record detailed data[4] on every stop. Below are data from all 2020 stops among the most common racial groups.[5]

Police found & reported criminal offense?

Race of Stopped Civilian

	Asian	Black	Hispanic	White	*Total
Yes	77	2431	824	352	3684
No	142	3765	1226	504	5637
*Total	219	6196	2050	856	9321

Two-Way Table: A table of counts describing two *categorical variables.

1. What are the two variables described in the table above?

   *Race of stopped civilian and whether police found and reported a criminal offense

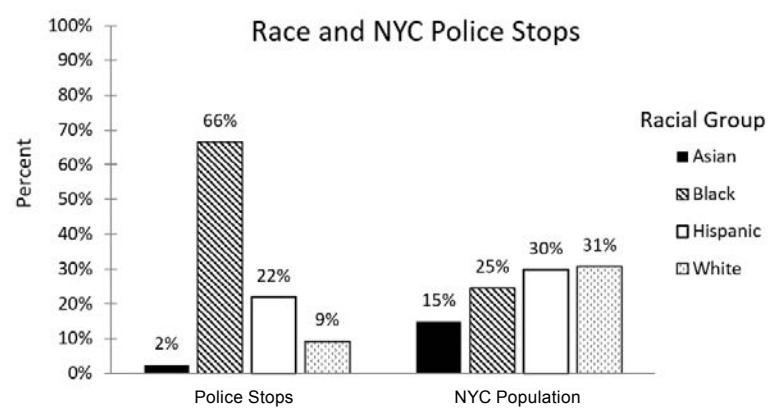
4 This NYPD data is publicly available on the NYC.GOV website: https://www1.nyc.gov/site/nypd/stats/reports-analysis/stopfrisk.page

5 Note: Groups not included (Middle Eastern, Native American, Not Reported) did not have a high enough sample size for this analysis. So, all percentages in this lesson are calculated only among the groups listed in the table. Data for other groups can be found in the NYC.GOV data file.

Marginal Distributions[6]

2. Find the racial distribution of civilians stopped by police.

* **Asian:** $\frac{219}{9321} = 2.3\%$ **Black:** $\frac{6196}{9321} = 66.5\%$ **Hispanic:** $\frac{2050}{9321} = 22.0\%$ **White:** $\frac{856}{9321} = 9.2\%$



3. What feature of this side-by-side bar graph is not needed to satisfy "title, tick, tick, label, label, label"?

   *The exact labels of the percents over the bars (this is shown just to help with later questions)

4. Compare the racial distribution of stops to the racial distribution of New York City. What do you notice? What do you wonder?

   *Police tend to stop Black civilians at a disproportionately high rate. They are the only group who are stopped at a higher rate than the rate we'd expect if randomly sampling among the population. I wonder: Does this have to do with crime rates?

6 NYC population data from 2020 American Community Survey (U.S. Census Bureau)

Conditional Distributions and Associations

Former NYC Mayor Michael Bloomberg, who expanded Stop and Frisk, speaking about the program (Aspen Institute, 2015):

> *"People say, 'Oh my God, you are arresting kids for marijuana who are all minorities.' Yes, that's true. Why? Because we put all the cops in the minority neighborhoods. Yes, that's true. Why'd we do it? Because that's where all the crime is."*

Today, advocates for Stop and Frisk make similar claims. Are these claims supported by the data? Let's investigate.

5. Calculate the crime rates found among each racial group.
   Display this information using a **segmented bar graph**.

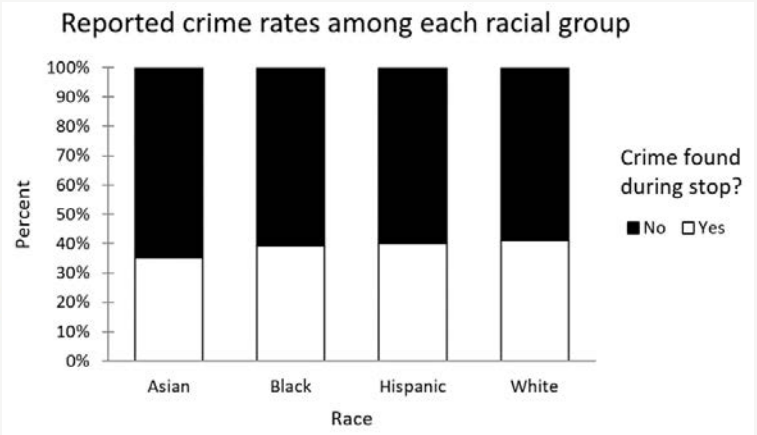
   * **Reported crime rates among …**

   Asian civilians: $\frac{77}{219} = 35.2\%$

   Black civilians: $\frac{2431}{6196} = 39.2\%$

   Hispanic civilians: $\frac{824}{2050} = 40.2\%$

   White civilians: $\frac{352}{856} = 41.1\%$



6. Do the data suggest an association between race and crime rates? Explain.

   *There appears to be a slight association between race and the rates of criminal activity found during stops. For example, police discovered criminal activity among 41.1 percent of stopped White civilians, which is a higher rate than among stopped Asian (35.2%) civilians.

Note: What are the three key components you should mention when describing an association?

- *Make a claim (there is/isn't an association)
- *Support claim by comparing percentages between groups
- *Include context (mention variables)

7. Are Mayor Bloomberg's comments supported by this data? Explain.

   *No—Stops uncovered similar rates of criminal activity among each racial group. In fact, stops among White civilians uncovered criminal activity at a slightly higher rate than the rest of the groups.

***Teacher Note:** This data can't definitively prove or disprove Bloomberg's claims. For example, the rates of crime discovered by police stops may differ from the rates of undiscovered (but still real) crime. Still, the mayor made these claims during a speech on "Stop and Frisk", and this data from the program (without making outside assumptions) falls short of supporting those claims.

Discourse in Math Class

A guide for discussions that utilize statistics, data science, and mathematics

Level 1: Discussion Norms

1. Debate the point, not the person.
2. Ask about intentions, speak about impact.
3. Back arguments with evidence—preferably data.

   Note: Data can't invalidate personal experiences.
4. One speaker at a time.

Level 2: Focused Speaking and Listening

1. Use classmates' names and refer to their points:
   a. "I agree with Destiny's thinking about … because …"
   b. "I would like to zoom in on Alejandro's point about …"
   c. "Although I see Nathan's point, I disagree because …"
   d. "I agree with Vanessa's point, and I believe we should also think about …"
2. Use complete sentences.
3. Be **specific**.
4. Take notes and give respondents ample time to respond.

Level 3: Question for Clarity and Learning

1. Good questions
   a. avoid yes/no responses;
   b. empower people to reflect, respond, and/or learn; and
   c. challenge assumptions.
2. When clarifying incomplete answers, ask questions like the following:
   a. "Please say more …" "Can you be more specific?"

b. "How would you explain that to someone who doesn't have a formal understanding of statistics?"
c. "Can you outline how that confounding factor explains the trend in both variables?"

3. When challenging assumptions, ask questions like the following:
   a. "Michael, can you walk me through how you came to that conclusion?"
   b. "What would happen if that assumption was false and the experimental units were instead chosen randomly?"
4. When helping someone start or complete their thoughts, ask questions like the following:
   a. "What is your first thought or instinct?"
   b. "What is the specific connection between … and …?"
5. When seeking to understand and learn, ask questions like the following:
   a. "How did you know to add rather than subtract the variances?"
   b. "Can you take me through how you reached that conclusion?"

Level 4: Lead and Shape Discourse

1. Ask classmates for clarification on their points:
   a. "Destiny, I was with you up until your point about outliers, but I did not understand after that. Can you clarify?"
   b. "So that we're all on the same page, we should first define what we mean exactly by the word 'correlation' …"
2. Summarize important points for consensus:
   a. "If I'm understanding correctly, we have now established that this study doesn't prove causation. So …"
   b. "I'm noticing a trend in the comments so far that …"
3. Refocus:
   a. "That's an interesting point, but I think we've strayed from the question at hand, which was …"
   b. "I think we can all agree on that point, but we haven't resolved the separate issue of …"
4. Bring in other perspectives:
   a. "No one has expressed this viewpoint yet, but I've heard others make the argument that …"

Appendix D.
Sample Mathematics Lesson for Essential Characteristic 4: Centers Community Knowledge

This lesson was created by a teacher during a design lab focused on culturally responsive and sustaining education (CRSE). The lesson aligns to Essential Characteristic 4, which is described in Table D1.

Table D1. Overview of Essential Characteristic 4

	Details
Essential characteristic	Characteristic 4: Centers community knowledge
Description of essential characteristic	Math curricula challenge dominant narratives about math and school, elevating historically marginalized voices.
Core feature that this lesson relates to	Curriculum materials challenge traditional ways of knowing/doing math (e.g., right/wrong binary, base-10, emphasis on individual achievement) by providing students opportunities for open-ended, problem-solving, social connection, and connecting math to their context.

Table D2 lists the mathematical standards to which this lesson aligns.

Table D2. Mathematical Standards Related to This Lesson

Component	Details
Source	Source: New York State Mathematical Standards
Domain	Understand that the digits of a three-digit number represent amounts of hundreds, tens, and ones.

Component	Details
Mathematical Standard	**NY-2.OA.1a** Use addition and subtraction within 100 to solve one-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions (e.g., using drawings and equations with a symbol for the unknown number to represent the problem). **NY-2.OA.1b** Use addition and subtraction within 100 to develop an understanding of solving two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions (e.g., using drawings and equations with a symbol for the unknown number to represent the problem).

Lesson Title: Counting Groups of Ten

Source material: This lesson is adapted from Lesson 5.6: Challenging Ableist Assumptions in Mathematics Problems in the following source: Koestler, C., Newton, J. R., & McGarry, J. (2022). Mathematics lessons for building and examining identities. In T. G. Bartell, C. Yeh, M. D. Felton-Koestler, & R. Q. Berry III (Eds.) Upper elementary mathematics lessons to explore, understand, and respond to social injustice. Corwin Press.

Lesson Goals

This lesson explores the issue of human diversity (specifically, different kinds of bodies), disability, and ableism. This lesson is meant to be launched during or after a lesson found in many textbooks that assumes students all have "typical" bodies, such as having 10 fingers, and are able to participate in "typical" ways. Students can use critical literacy skills to examine this type of mathematics lesson and associated resources used during the lesson to see how bodies, disability, and ableism are presented.

Oftentimes, the topic of disability in mainstream classrooms is invisible or not talked about explicitly unless absolutely necessary. It is important for students to see both children and adults with disabilities represented in their classrooms in empowering ways. Disabilities should be portrayed in ways that avoid deficits and stereotypes and instead accurately describe the disability and/or portray people with disabilities living their lives (whether or not disability is the focus).

This lesson engages students in using their bodies—specifically, their fingers—as a physical representation to support skip-counting groups of 10. At the same time, students will unpack this common practice to begin a conversation about body diversity.

Lesson Setup

Table D3 provides some key information to help teachers set up and carry out the lesson.

Table D3. Lesson Setup

	Details
Lesson materials	• Chalkboard/whiteboard • Chalk or markers
Books for classroom library (optional)	• *All Are Welcome* by Alexandra Penfold • *The Bug Girl* by Sophia Spencer • *Emmanuel's Dream: The True Story of Emmanuel Ofosu Yeboah* by Laurie Ann Thompson and Sean Qualls • *Hello Goodbye Dog* by Maria Gianferrari • *I Am Not a Label* by Cerrie Burnell • *A Kids Book About Disabilities* by Kristine Napper • *Mama Zooms* by Jane Cowen-Fletcher • *Rescue and Jessica: A Life-Changing Friendship* by Jessica Kensky and Patrick Downes • *Terry Fox and Me* by Mary Beth Leatherdale • *What Happened to You?* by James Catchpole and Karen George

Lesson Procedures: Day 1

Launch (30 minutes)

» A common elementary mathematics activity is to count by tens by counting all the fingers in the classroom. This activity makes an ableist assumption that all people have 10 fingers. This lesson engages students in questioning those assumptions and thinking about different mathematical contexts we can use for making tens. Prior to implementing this lesson, we recommend you consider your classroom and how you want to frame the topic. This is important for all classes, but it is especially critical if you have any students who do not have 10 fingers. In the section below, *Elevating CRSE in the Lesson*, there are several links that can be helpful in thinking about how to discuss these topics with your students. If you have a student(s) who does not

have 10 fingers, we suggest you discuss the lesson with the family and child to make sure you are approaching it in a way that feels inclusive and supportive to the student(s). While we provide suggestions for some possible approaches to navigating this space, you should adjust based on your context. Say something like this to students:

> I have been thinking a lot about today's activity and wanted to talk to you about it. It is an activity that is in a lot of textbooks because it is usually really good at getting kids to think about patterns in our number system, but I also am wondering about some assumptions it makes about kids and bodies. Let's look at the task and think about some of the hidden assumptions it makes before we start.

» Say to students: "The activity that is usually in textbooks is to figure out the total number of fingers in our class by counting by tens." (You may want this displayed on the board as well.) If students need additional support, you may want to explicitly ask the following:

- "What does this problem assume about people's bodies? About their fingers?"
- "How do you know the problem assumes this?"

» Have students share their responses and ask the class: "Does everyone in our world have 10 fingers? Why or why not?"

- Students may have examples of people in their lives who were born with a different number of fingers or who lost a finger(s) during their lifetime. If not, explain that this could happen. If you have a student who does not have 10 fingers in your class, be mindful of not expecting them to speak as a "representative" for those with a different number of fingers and of not allowing other students to do the same.

» At this point, we offer two possible ways to move forward in the lesson. You may choose to continue with the activity as it is commonly stated (count by tens to find the total number of fingers in the class), or you may ask the class to brainstorm other things that come in tens that you can use for the problem (we especially recommend this second approach if someone[s] in the class does not have 10 fingers). You might tell the class: "The mathematics goal for this problem is to count by tens, but since not everyone has 10 fingers, let's think of some other things that could come in tens that we can count by instead." The class might suggest ideas like boxes of markers, bags of marbles, or the tens sticks from base-10 blocks.

- Choose one of these and prepare one for each person in the class plus several extras before continuing the lesson. Alternatively, you can have the base-10 sticks available and suggest to the class that you use these to represent whatever group of 10 objects the class has agreed on. Give each student one of the tens.

» Tell the class, "We're going to count by tens to see how many _____ we have in our whole class."

» Have students raise their hands or objects as they say their numbers (10, 20, 30 . . .). You can keep track of the count by listing the numbers, drawing a number line, or using other ways your students suggest to record the numbers.

Teacher Note

You should adjust what you would say depending on your context. For example, if you have already done work with critical literacy, your students may be familiar with the idea of how authors' assumptions and biases can be analyzed. If not, you may have to discuss it a bit more, perhaps by asking if they know what an assumption is and if they can give any examples. For more information about critical literacy, see Vivian Maria Vasquez's (2016) book, *Critical Literacy Across the K–6 Curriculum*.

Explore (30–45 minutes)

» Try to launch the activity more than once, starting with different students. Ask students to describe what they notice. Ask students questions about what would happen in different scenarios. Here are some examples:

- "If there are any students absent from class, what would be the number of [objects] if they were present?"
- "What if the art, music, and physical education teachers joined the class?"
- "What if we had 35 students in our class?"
- "What if we counted the whole third grade, which has 78 students?"

» Be sure to entertain multiple strategies for solving the tasks (e.g., counting by tens, adding on by the multiple of 10). As you work, record the problems and solutions (e.g., 18 × 10 = 180) in a vertical list off to the side or on chart paper that you can refer back to in the next lesson when the class will look for patterns.

Summarize (20 minutes)

» Briefly summarize the mathematical strategies that students used or ask them to identify any common strategies. Then, say that you'll be exploring problems with tens more next time. Next, read one of the suggested children's books. It may be good to have several books to choose from and to begin a study of how disability is explored similarly and differently in the books. We suggest that you continue to have books like those suggested in the *Resources and Materials* in your classroom throughout the school year, so that students see these are a part of the classroom library and not just as part of a lesson on disability.

» While you are reading and/or once you are done reading, have students share questions they have about differences and disabilities.

Lesson Procedures: Day 2

Launch (20 minutes)

- Remind the class about what they learned in the previous day's lesson, both about questioning the hidden assumptions in mathematical problems and your mathematical strategies. Explain that today you will focus more on the mathematical strategies and in a future lesson you will return to exploring hidden assumptions and inclusion in mathematics problems.
- The list of problems from Lesson 1 (e.g., 18 × 10 = 180) should be displayed to the side of the board—but do not focus on it yet. Pose some questions like the following to the class: "How many tens does it take to make 780? How do you know?"
- Review the meaning of the place value for a number like 780. "What does each digit stand for?" Point out that the 7 stands for 7 *hundreds* and ask them how many tens we can think of this as. Point out the connection to the previous problem: "780 stands for 7 hundreds and 8 tens, but it can also be thought of as 78 tens."
- Review the list of problems solved from Lesson 1 (e.g., 18 × 10 = 180) and the list of problems solved today. Ask the students if they notice any patterns. You can also ask more explicitly: "What happens when we multiply a number by 10?" Students will likely notice that you "add" a 0 to the end of the number. You should question or clarify the use of the word "add" and point out that they don't mean, for example, 18 + 0. You can suggest that another word they can use is "append" or that we might think of this as moving the digits one place to the left (i.e., ones move to the tens place, tens move to the hundreds place, and so on).
- Write this as a class conjecture. If students are not familiar with that term, explain that it's something that you think is true in mathematics, but you need to explore it more to determine if it's always true or not. Tell the class that today they will be working with examples, base-10 blocks, and pictures to try to determine if this is always true and to then prove that it's true.

Explore (20 minutes)

- Students work in pairs or small groups to explore the conjecture. They may want to begin with several examples, then ask them to think about how they can show that it would work with any numbers. Encourage them to think about how they can use base-10 blocks and/or drawings to show what is happening and why that makes this true. Question groups about how they can use what they learned from the warm-up to help them on this problem (e.g., that 780 has 78 tens).
- It may help to have students first think about single-digit numbers (e.g., 7 × 10) before thinking about double-digit numbers. Circulate among the groups, see what they are thinking, ask probing questions, and make decisions about what explanations to have students share and in what order.

Optional Extensions

It is likely that most students will stick to whole numbers. Depending on the progress and understanding of the class, you may choose to challenge individual groups or the whole class with different types of numbers such as fractions or decimals. This can be helpful for a conversation about how to modify and/or limit conjectures. The conjecture could be limited to whole numbers, or it could be modified to include decimals by being precise about shifting digits one place value to the left as opposed to appending a zero. The conjecture does not apply to fractions. You can also explore what happens when we **divide** a number by 10.

Summarize (30 minutes)

- Have some or all groups share their thinking about why multiplying by 10 results in appending a zero to the end of the number. Before starting, tell students that when they listen to groups share, they should think of compliments (i.e., something they found valuable about the group's mathematics) and questions (i.e., things they didn't understand or want to understand better). The class should engage in a discussion about the different explanations.
- End by highlighting the key strategies that students used and emphasizing that mathematicians often spend a long time developing a clear proof that something is true, so it's not something you usually figure out in one day. Depending on the progress made, you may choose to revisit this concept in the future.
- Explain that next time you will return to a focus on the hidden assumptions in mathematics problems.

Optional: Taking Action

As an ongoing investigation, have students examine ways in which people are portrayed in the books in your classroom, including in your mathematics curricular materials.

For example, you may compare how people with disabilities are represented in *All Are Welcome* by Alexandra Penfold (i.e., where the children just happen to be using a wheelchair or a white cane but not specifically discussed as having disabilities) versus in *Emmanuel's Dream: The True Story of Emmanuel Ofosu Yeboah* by Laurie Ann Thompson and Sean Qualls (i.e., where a true-life story is illuminated by exploring what it was like growing up with a disability).

Students can note places in their textbooks where there are assumptions that everyone in the classrooms is the same, especially in terms of being able-bodied. They may choose to take action by writing letters to different audiences, such as the textbook publishers or district administrators (curriculum coordinators), describing what they observe as inaccurate depictions of people in their classroom and/or world and providing suggestions of ways to make the content and materials more inclusive and representative of diversity. While this activity is ideally student led, students may need some assistance in developing suggestions to improve diverse representation in the materials.

Elevating CRSE in the Lesson

Before teaching this lesson, you should reach out to families, parents/caregivers, and administrators in your building to provide an overview of the topics included in this lesson (different kinds of bodies, disability, and ableism) and the kinds of discussions that might emerge. This will help you anticipate ways to be sensitive to and inclusive of the students in your classroom. Any information you receive about specific students or their family members (about differences or disabilities) should stay private, unless they give you explicit permission to share. As mentioned earlier in the lesson, take care not to place any student(s) in a position where they have to speak as a "representative" for those who are different or who are disabled.

Other resources include the following:

- Article: "How to talk to your kid about disabilities," by Caroline Bologna, *Huffington Post*, March 1, 2021
- Lesson: Learning for Justice, "Picturing Accessibility: Art, Activism and Physical Disabilities"
- Lesson: Learning for Justice, "What Is Ableism?"
- Lesson: Learning for Justice, "What Is a Disability?"

The lesson is about counting tens and introducing the idea that many students count using their fingers, which might not be possible for all students as not all people have 10 fingers. The CRSE element is to help students develop the skill to question assumptions—which in this case is the as the ableist assumption that everyone has 10 fingers to count on. During the opening of a lesson, many students may be reluctant to openly share their experiences around bullying from other classmates, sexism, racism, and so on out of fear of how they may be perceived by their peers. Similarly, students are less apt to share their mathematical thoughts without opportunities for self-reflection and peer checking. Teachers may purposefully group students in ways that provide support for student sharing of ideas.

During the exploration portion of a lesson, you should monitor student progress both mathematically and socially. Consider

- purposefully monitoring student explorations and facilitating these pathways toward meaningful mathematical and social discoveries,
- completing brief notes on students' mathematical pathways to resolving the social injustice, and
- looking for students who have stories to tell that relate to the social justice math lesson and providing them with the opportunity to share with others outside of their small group.

Appendix E.
Sample Mathematics Lesson for Essential Characteristic 5: Promotes Critical Mathematical Thinking and Learning

This lesson was created by a teacher during a design lab focused on culturally responsive and sustaining education (CRSE). It focuses on a core feature of Essential Characteristic 5, described in table E1.

Table E1. Overview of Essential Characteristic 5

	Details
Essential characteristic	Characteristic 5: Promotes critical mathematical thinking and learning
Description of essential characteristic	Math curricula hold high expectations for student learning, supporting the development of positive math identities.
Description of essential characteristic	• Curriculum materials are aligned with relevant state and/or national standards and ensure that students will acquire the knowledge and skills needed for academic success. • Students have multiple, diverse opportunities to display and communicate their thinking. • Students have the opportunity to reason, problem-solve, and make real-world connections. • Curriculum materials encourage student questioning and ownership of their learning.

Table E2 lists the mathematical standards to which this lesson aligns.

Table E2. Mathematical Standards Related to This Lesson

Component	Details
Source	Common Core State Standards
Domain	Geometry

Component	Details
Mathematical Standard	CCSS.MATH.CONTENT.HSA.APR.C.4 Prove polynomial identities and use them to describe numerical relationships. CCSS.MATH.CONTENT.HSG.GPE.B.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

Lesson Title: Healthy Food for Healthy People—Addressing Food Deserts and Food Insecurity in BIPOC Communities

This lesson is taken from *Pollyanna Racial Literacy Curriculum: High School Math*, which is available for free on the Pollyanna 9–12 Racial Literacy Curriculum web page.

Lesson Goals

Social Justice Standards learning outcomes:

- DI.9-12.8: I respectfully express curiosity about the history and lived experiences of others and will exchange ideas and beliefs in an open-minded way.
- JU.9-12.12: I can recognize, describe and distinguish unfairness and injustice at different levels of society.

This lesson explores the pervasiveness of food deserts across the United States, focusing specifically on the lack of access to affordable, healthy foods for people living in low-income BIPOC (Black, Indigenous, and People of Color) communities.

- Day 1 includes two activities that help students explore these issues by making sense of data that has been disaggregated by race.
- Day 2 asks students to apply a racial-equity lens while using mathematical concepts to determine where a new supermarket should be built.
- Day 3 focuses on one strategy to address the problem of food deserts—the construction of a community garden—and engages students in an open-ended, "low-floor, high-ceiling" math task that can be adapted for a wide range of learners.

Lesson Setup

Table E3 provides some key information to help teachers set up and carry out the lesson.

Table E3. Lesson Setup and Logistics

Aspect of Lesson Setup	Details
Lesson source	Osler, J. (2022). Math lesson 2: Healthy food for health people—Addressing food deserts and food insecurity in BIPOC communities. In *Pollyanna Racial Literacy Curriculum: High School Math*. Pollyanna, Inc. https://b51f3f.p3cdn1.secureserver.net/wp-content/uploads/2022/05/High-School-Curriculum-Math.pdf
Key vocabulary	• Census tract • Food desert • Food insecurity • Distance formula
Lesson objectives	• Students will write and compare rates to assess disproportionate impact. • Students will apply the distance formula to determine the distance between two points on a cartesian plane. • Students will maximize the area of a rectangle given a fixed perimeter. • Students will recognize unfairness on the individual level (e.g., biased speech) and injustice at the institutional or systemic level (e.g., discrimination). • Students will respectfully express curiosity about the history and lived experiences of others and will exchange ideas and beliefs in an open-minded way.

Lesson Procedures

Day 1: Food Deserts in BIPOC Communities

1. Introduction and reflection: "Why is eating healthy food important? Where do people go to get healthy food? Does everyone have access to healthy food, or are there some communities that have more or less access than others? And when communities lack access to affordable healthy food, what can they do to address this problem? These are some of the questions we will be exploring in this lesson."

2. Have students brainstorm a list of places their families can go to purchase healthy foods. For urban communities, make a list of places within one mile (within approximately 20 minutes of walking); for rural communities, list places within 10 miles. Examples could include grocery stores, supermarkets, farm stands, farmers markets, and big-box stores like Target that sell produce. This list should not include fast-food restaurants, corner stores, or convenience stores. Compile a class list if there are a variety of places students identify. Have students reflect on the extent to which they live in a community that has geographic proximity, and thus easy access, to stores that sell healthy food.
3. Show students the video "The Food Deserts of Memphis: Inside America's Hunger Capital" (available on page 13 of *Pollyanna Racial Literacy Curriculum: High School Math*). Students should take notes while watching the video; consider asking students to write down three important facts or big ideas as they watch.
4. Facilitate a brief discussion in small groups or as a class. Questions to pose include the following:
   - What is a food desert?
   - Where do people living in food deserts get food, and what kinds of food do they have the easiest access to?
   - What is it like, or how do you think it might feel, to live in a food desert?
   - How might living in a food desert impact one's health and well-being?
5. The next activity will help students understand that communities of color are disproportionately impacted by lack of access to affordable, healthy foods. Distribute or project the student handout entitled "Hunger in U.S. Families With Children, May 2020–March 2021" (available on page 18 of *Pollyanna Racial Literacy Curriculum: High School Math*). In pairs, small groups, or as a whole class, have students respond to the following prompts:
   - What do you notice? If you make a claim, state what you noticed that supports your claim.
   - What do you wonder? What are you curious about that comes from what you notice in the graph?
   - What is going on in this graph? Write a catchy headline that captures the graph's main idea.
   - What do you notice when comparing hunger rates between Black and Latinx families on the one hand, and White and Asian American families on the other? How could you describe these differences using math terminology? What are some different ways you could use math language to describe the differences? (Consider having students find the total difference between the averages, calculate the percent difference between racial groups, compare one set of data as a percent of the other, and/or describe how many times greater one group's hunger levels are than another's.)
   - What impact do federal unemployment benefits seem to have on hunger levels? Why would this be?
6. Distribute the handout called "Supermarket Access by Race and Poverty Level in the United States" (available on page 19 of *Pollyanna Racial Literacy Curriculum: High School Math*). Have students work independently or with a partner to answer the three observation questions, and then facilitate a whole-class discussion so that students can hear one another's observations, questions, and headlines.

7. Provide students with time to answer additional questions on the handout; you will have to decide if you want to assign some or all of these questions or to add your own.
8. Assess students' learning by asking them to summarize, in writing, what they think are the most important takeaways from the activities and to use mathematical evidence to justify or explain their perspectives. This could be in the form of an exit ticket or a homework assignment.

Day 2: Applying a Racial Equity Lens to Building a Supermarket

1. Remind students that, in the previous class, they learned about food deserts and saw that low-income Black and Latinx neighborhoods often have less access to supermarkets than low-income White neighborhoods. Inform students that today they are going to apply this information as they determine the fairest location to build a new supermarket.
2. Distribute the handout called "Building in the (Food) Desert" (available on page 18 of *Pollyanna Racial Literacy Curriculum: High School Math*). Have students work in pairs or small groups to apply the distance formula to determine the distance between Neighborhood A and the Black Star Supermarket (Question 1 in the handout).
3. After students have completed Question 1, use a brief, whole-class discussion to check for understanding and address any confusion or misconceptions.
4. Have students complete the remainder of the assignment. When students begin looking for a location to place the Black Star Supermarket, there are several things they should consider. You can use the following prompts to scaffold the activity for struggling learners, require groups that finish quickly to justify their analysis using additional mathematical concepts, or challenge groups to think more equitably about their solution.
   - The Red Star Supermarket is closest to Neighborhood C (predominantly White). Ask students if they think the new market should be built an equal distance from each neighborhood. Is that the fairest approach? Why or why not?
   - If students decide to build the Black Star Supermarket closer to Neighborhoods A and/or B, how far away from Neighborhood C would they feel comfortable placing it?

   This task might be different if the Red Star Supermarket did not exist; in that case, students might feel justified placing the Black Star Supermarket at a point equidistant from the three neighborhoods. But this is not the case, because there is already one supermarket in Center City.

   - How should students factor in the existing access (or lack thereof) between each neighborhood and the Red Star Supermarket?
   - How might students justify placing the Black Star Supermarket directly between Neighborhoods A and B? How would they determine this location on the map?

- While the task does not ask this of students, they could calculate the average distance between each neighborhood and the two supermarkets. Knowing this average could help them justify a location for the new market. Would it be fairest to place the new market where the average distance between a neighborhood and the two supermarkets is the same? Or, given what students have explored about food insecurity disproportionately impacting Black and Latinx communities, would it be fairer for the average distance to the two supermarkets to be lower for Neighborhoods A and B?

5. Provide students with an opportunity to share their final results, along with their justification for where they decided to build the new supermarket, including their mathematical calculations.

Day 3: Maximizing the Area of a Community Garden

1. Let students know that some communities facing food insecurity and lacking access to healthy foods have taken matters into their own hands by starting grocery cooperatives, creating neighborhood gardens, and distributing food directly to people. In Oakland, California, there is a neighborhood called West Oakland, a historically Black neighborhood that is now home to many low-income communities of color; until recently, this neighborhood did not have any supermarkets. Inform students that they will watch a short video about a community organization in West Oakland that developed a way to get healthy food to local residents.
2. Show students the video "Uprooting Food Injustice in West Oakland" (3:07 minutes, available on page 15 of *Pollyanna Racial Literacy Curriculum: High School Math*). While students watch the video, have them complete a 3-2-1 chart that includes three facts, two opinions, and one big idea from the video.
3. Distribute the handout called "City Slicker Garden" (available on page 23 of *Pollyanna Racial Literacy Curriculum: High School Math*). Have students, in pairs or small groups, determine the area of the two rectangles based on the diagram provided in the handout. After a few minutes, provide students with the opportunity to explain how they calculated the area of the two rectangles, so that everyone understands the problem they are being asked to solve and has a plan for how they will look for an answer.
4. Provide students with time to work toward a solution. Students may need to complete their work asynchronously as homework.
5. End class by engaging students in a discussion about their findings. Have different students or groups of students share their answers, and consider posing some of the following questions as you facilitate the conversation:
   - How would you determine which of these answers is best?
   - Why did different groups come up with different solutions?
   - What was a strategy you saw a different person or group using that you liked? Why do you think this was an effective strategy?
   - Did anyone make a table of their results? If so, what did it look like?
   - What do you think a graph of these results would look like?

Elevating CRSE in This Lesson

This lesson is also a great example of *Essential Characteristic 3: Integrates Socially Responsible, Real-World Contexts* in that it requires students to grapple with a modern-day injustice—lack of affordable food options in areas with "food deserts." The first day requires students to brainstorm a list of food options. Teachers should come prepared with a list to help guide students' thinking. Teachers should also be prepared for students to potentially have a reaction to learning that they live in an area that is considered a food desert and be prepared with ideas for them to challenge this injustice should they want to do so.